D01 FORCES, FREE-BODY DIAGRAMS & NEWTON'S LAWS OF MOTION

SPH4U

CH 2 (BIG PICTURE)

- define and describe concepts and units from the analysis of forces
- distinguish between accelerating and nonaccelerating frames of reference
- determine the net force acting on an object and its resulting acceleration by analyzing experimental data using vectors and their components, graphs, and trigonometry
- analyze and predict, in quantitative terms, and explain the cause of the acceleration of objects in one and two dimensions
- analyze the principles of the forces that cause acceleration, and describe how the motion of human beings, objects, and vehicles can be influenced by modifying factors

EQUATIONS

- Newton's Second Law of Motion

$$\Sigma\vec{F} = m\vec{a}$$

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

- Weight (Force due to Gravity)

$$\vec{F}_g = m\vec{g}$$

FORCES

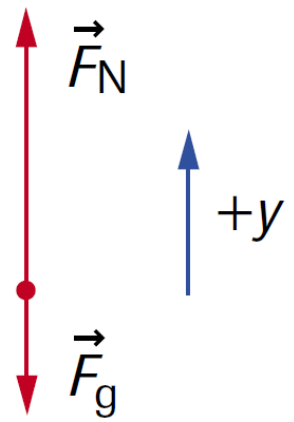
- **Force ($\vec{F}$) [**N = kg m s^{-2}**]:** a push or a pull
- Common Forces
  - Force of gravity ($\vec{F}_g$): force of attraction between all objects
  - Normal force ($\vec{F}_N$): force perpendicular to the surfaces of objects in contact
  - Tension ($\vec{F}_T$): force exerted by materials, such as ropes, fibres, springs, and cables, that can be stretched

FORCES – CONT.

- More Common Forces:
  - Friction ($\vec{F}_f$): force that resists motion or attempted motion between objects in contact; acts in direction opposite to motion or attempted motion
    - Static Friction ($\vec{F}_S$): force that tends to prevent a stationary object from starting to move
    - Kinetic Friction ($\vec{F}_K$): force that acts against an objects motion
    - Air Resistance ($\vec{F}_{air}$): frictional force that opposes an objects motion through air
  - Applied Force ($\vec{F}_{app}$): any other force acting upon the object

DRAWING FREE-BODY DIAGRAMS

- **Free-Body Diagram (FBD):** a diagram of a single object showing all the forces acting on that object
  - Object represented by a dot, box, or small sketch
  - Coordinate system is shown (which ways are positive)
  - Arrows point in the direction of the force with an appropriate magnitude



EXAMPLE 1

You toss a ball vertically upward. Draw a FBD of the ball just before it leaves your hand.

EXAMPLE 1 – SOLUTIONS

Only two forces act on the ball (**Figure 5**). Gravity acts downward. The normal force applied by your hand (we may call this the applied force, since it comes from you) acts upward. Since there are no horizontal components of forces in this situation, our FBD shows a $+y$ direction, but no $+x$ direction.

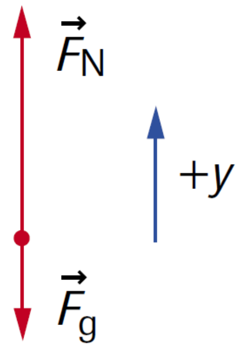


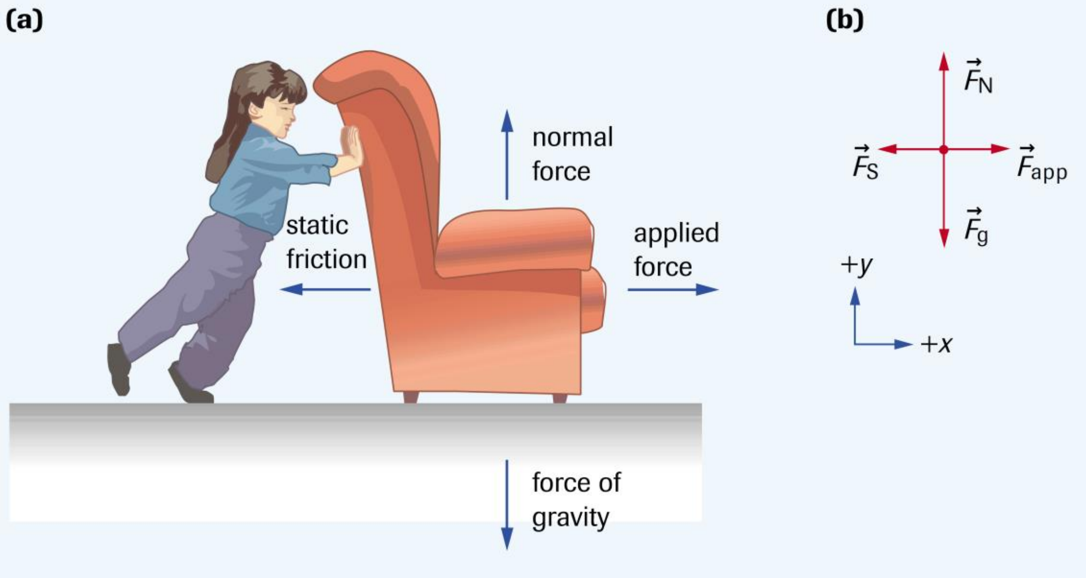
Figure 5
The FBD of the ball in Sample Problem 1

EXAMPLE 2

A child is pushing with a horizontal force against a chair that remains stationary. Draw a system diagram of the overall situation and an FBD of the chair.

EXAMPLE 2 – SOLUTIONS

The system diagram in **Figure 6(a)** shows the four forces acting on the chair: gravity, the normal force, the applied force (the push delivered by the child), and the force of static friction. The $+x$ direction is chosen in the direction of the attempted motion. **Figure 6(b)** is the corresponding FBD, showing these same four forces.

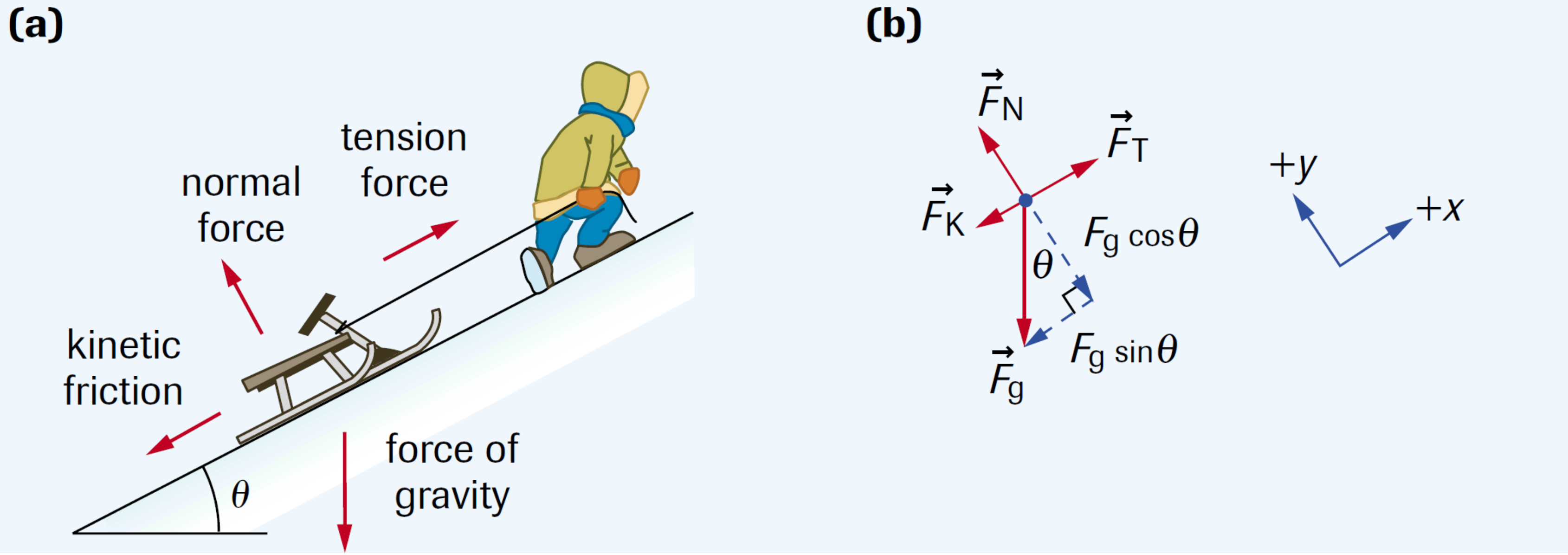


EXAMPLE 3

A child pulls a sleigh up a snow-covered hill at a constant velocity with a force parallel to the hillside. Draw a system diagram of the overall situation and an FBD of the sleigh.

EXAMPLE 3 – SOLUTIONS

The system diagram of **Figure 7(a)** shows the four forces acting on the sleigh: gravity, tension in the rope, kinetic friction, and the normal force. The $+x$ direction is the direction of motion and the $+y$ direction is perpendicular to that motion. **Figure 7(b)** is the corresponding FBD, including the components of the force of gravity.



FORCES ON STATIONARY OBJECTS

- Net Force ($\Sigma\vec{F}$): the sum of all the forces acting on an object

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

EXAMPLE 4

In hitting a volleyball, a player applies an average force of 9.9 N [33° above the horizontal] for 5.0 ms. The force of gravity on the ball is 2.6 N [down]. Determine the net force on the ball as it is being struck.

EXAMPLE 4 – SOLUTIONS

The relevant given information is shown in the FBD of the ball in **Figure 9(a)**. (Notice that the time interval of 5.0 ms is not shown because it is not needed for this solution.) The net force on the ball is the vector sum $\vec{F}_g + \vec{F}_{app}$. We calculate the net force by taking components with the $+x$ and $+y$ directions as in **Figure 9(b)**.

First, we take components of $\vec{F}_{app}$:

$$F_{app,x} = (9.9\ \text{N})(\cos 33°) \qquad F_{app,y} = (9.9\ \text{N})(\sin 33°)$$
$$F_{app,x} = 8.3\ \text{N} \qquad F_{app,y} = 5.4\ \text{N}$$

Next, we take components of $\vec{F}_g$:

$$F_{gx} = 0.0\ \text{N} \qquad F_{gy} = -2.6\ \text{N}$$

EXAMPLE 4 – SOLUTIONS CONT.

We add the components to determine the net force:

$$\sum F_x = F_{\text{app},x} + F_{gx} \qquad \sum F_y = F_{\text{app},y} + F_{gy}$$

$$= 8.3\ \text{N} + 0.0\ \text{N} \qquad = 5.4\ \text{N} + (-2.6\ \text{N})$$

$$\sum F_x = 8.3\ \text{N} \qquad \sum F_y = 2.8\ \text{N}$$

Figure 9(c) shows how we determine the magnitude of the net force:

$$\left|\sum \vec{F}\right| = \sqrt{(8.3\ \text{N})^2 + (2.8\ \text{N})^2}$$

$$\left|\sum \vec{F}\right| = 8.8\ \text{N}$$

The direction of $\sum \vec{F}$ is given by the angle ϕ in the diagram:

$$\phi = \tan^{-1} \frac{2.8\ \text{N}}{8.3\ \text{N}}$$

$$\phi = 19^\circ$$

The net force on the ball is 8.8 N [19° above the horizontal].

NEWTON'S LAWS OF MOTION

- **Newton's First Law of Motion:** If the net force acting on an object is zero, that object maintains its state of rest or constant velocity.
  - The Law of Inertia
- **Inertia:** the property of matter that causes an object to resist changes to its motion
  - "an object at rest stays at rest"
  - "an object in motion stays in motion"
- **Equilibrium:** property of an object experiencing no acceleration
- If in 2 dimensions, resolve any vectors into their horizontal and vertical.

EXAMPLE 5

A 12-passenger jet aircraft of mass 1.6×10^4 kg is travelling at constant velocity of 850 km/h [E] while maintaining a constant altitude. What is the net force acting on the aircraft?

EXAMPLE 5 – SOLUTIONS

According to Newton's first law, the net force on the aircraft must be zero because it is moving with a constant velocity. **Figure 5** is an FBD of the aircraft. The vector sum of all the forces is zero.

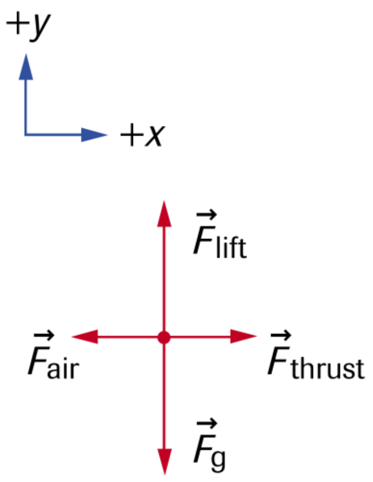


Figure 5
The FBD of the aircraft in Sample Problem 1

EXAMPLE 6

The traction system of **Figure 7** stabilizes a broken tibia. Determine the force of the tibia on the pulley. Neglect friction.

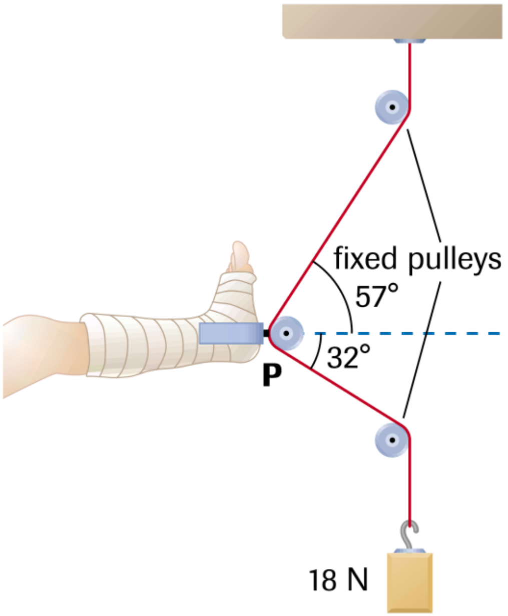


Figure 7
The system diagram of a leg in traction for Sample Problem 3

EXAMPLE 6 – SOLUTIONS

The FBD of the pulley (P) is shown in **Figure 8**. Since there is only one cord, we know there must be only one tension, which is 18 N throughout the cord. The net force on the pulley consists of both the horizontal (x) and vertical (y) components. We will use $\vec{F}_{\text{tibia}}$ as the symbol for the force of the tibia on the pulley.

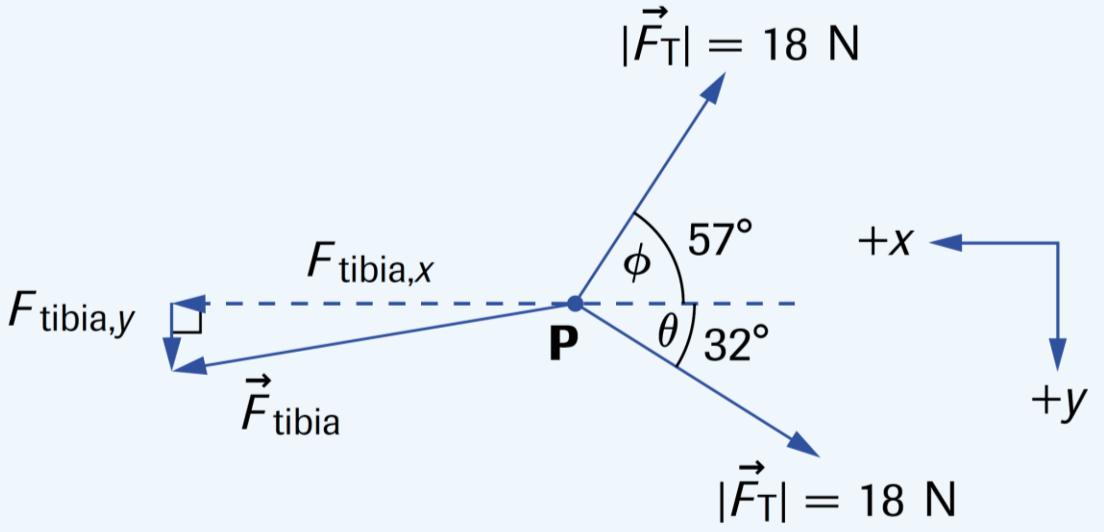


Figure 8
The FBD of the pulley (P)

EXAMPLE 6 – SOLUTIONS CONT.

Horizontally:

$$\begin{aligned}
\sum F_x &= 0 \\
F_{\text{tibia},x} - F_{\text{T}} \cos\phi - F_{\text{T}} \cos\theta &= 0 \\
F_{\text{tibia},x} &= F_{\text{T}} (\cos\phi + \cos\theta) \\
&= (18\text{ N})(\cos 57° + \cos 32°) \\
F_{\text{tibia},x} &= 25\text{ N}
\end{aligned}$$

Vertically:

$$\begin{aligned}
\sum F_y &= 0 \\
F_{\text{tibia},y} - F_{\text{T}} \sin\phi + F_{\text{T}} \sin\theta &= 0 \\
F_{\text{tibia},y} &= F_{\text{T}} (\sin\phi - \sin\theta) \\
&= (18\text{ N})(\sin 57° - \sin 32°) \\
F_{\text{tibia},y} &= 5.6\text{ N}
\end{aligned}$$

EXAMPLE 5 – SOLUTIONS CONT.

From **Figure 9**, we can calculate the magnitude of the force:

$$\left|\vec{F}_{\text{tibia}}\right| = \sqrt{(25\text{ N})^2 + (5.6\text{ N})^2}$$
$$\left|\vec{F}_{\text{tibia}}\right| = 26\text{ N}$$

We now determine the angle:

$$\omega = \tan^{-1}\frac{F_{\text{tibia},y}}{F_{\text{tibia},x}}$$
$$= \tan^{-1}\frac{5.6\text{ N}}{25\text{ N}}$$
$$\omega = 13^\circ$$

The force of the tibia on the pulley is 26 N [13° below the horizontal].

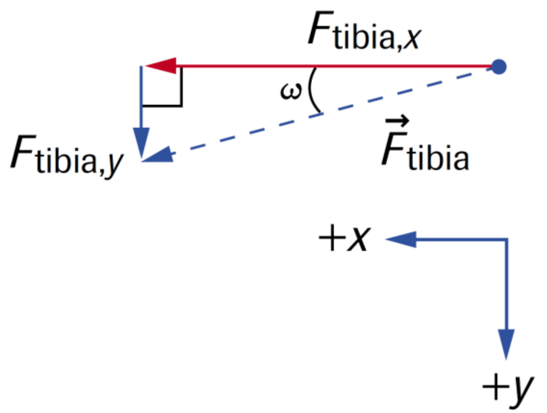


Figure 9
The force on the leg for Sample Problem 3

NEWTON'S LAWS OF MOTION

- **Newton's Second Law of Motion:** If the external net force on an object is not zero, the object accelerates in the direction of the net force. The acceleration is directly proportional to the net force and inversely proportional to the object's mass.

$$\Sigma\vec{F} = m\vec{a}$$

- For two dimensions:

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

EXAMPLE 6

The mass of a hot-air balloon, including the passengers, is 9.0×10^2 kg. The force of gravity on the balloon is 8.8×10^3 N [down]. The density of the air inside the balloon is adjusted by adjusting the heat output of the burner to give a buoyant force on the balloon of 9.9×10^3 N [up]. Determine the vertical acceleration of the balloon.

EXAMPLE 6 – SOLUTIONS

$$m = 9.0 \times 10^2 \text{ kg}$$
$$F_g = |\vec{F}_g| = 8.8 \times 10^3 \text{ N}$$
$$F_{app} = |\vec{F}_{app}| = 9.9 \times 10^3 \text{ N}$$
$$a_y = ?$$

Figure 11 is an FBD of the balloon.

$$\sum F_y = ma_y$$
$$a_y = \frac{\sum F_y}{m}$$
$$= \frac{F_{app} - F_g}{m}$$
$$= \frac{9.9 \times 10^3 \text{ N} - 8.8 \times 10^3 \text{ N}}{9.0 \times 10^2 \text{ kg}}$$
$$= \frac{1.1 \times 10^3 \text{ kg}\cdot\text{m/s}^2}{9.0 \times 10^2 \text{ kg}}$$
$$a_y = 1.2 \text{ m/s}^2$$

The acceleration of the balloon is 1.2 m/s^2 [up].

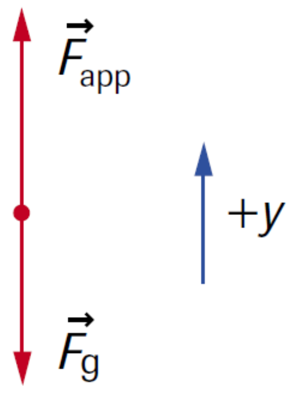


Figure 11
The FBD of the balloon shows the vertical forces as vectors. When only components are considered, the vector notation is omitted.

WEIGHT AND EARTH'S GRAVITATIONAL FIELD

- **Weight ($\vec{F}_g$):** the force of gravity on an object

$$\vec{F}_g = m\vec{g}$$

- **Force Field:** space surrounding an object in which a force exists
- **Gravitational Field Strength ($\vec{g}$):** amount of force per unit mass
  - $\vec{g} = 9.81\ \mathrm{m/s^2}$ [down]

NEWTON’S LAWS OF MOTION

- **Newton’s Third Law of Motion**: for every action force, there is a simultaneous reaction force equal in magnitude but opposite in direction.
  - action/reaction law
  - “every action has an equal and opposite reaction”

EXAMPLE 7

A softball player sliding into third base experiences the force of friction. Describe the action-reaction pair of forces in this situation.

EXAMPLE 7 – SOLUTIONS

We arbitrarily designate the action force to be the force of the ground on the player (in a direction opposite to the player’s sliding motion). Given this choice, the reaction force is the force exerted by the player on the ground (in the direction of the player’s sliding motion).

SUMMARY: FORCES AND FREE-BODY DIAGRAMS

- We commonly deal with Earth's force of gravity, the normal force, tension forces, and friction forces.
- Static friction tends to prevent a stationary object from starting to move; kinetic friction acts against an object's motion. Air resistance acts against an object moving through air.
- The free-body diagram (FBD) of an object shows all the forces acting on that object. It is an indispensable tool in helping to solve problems involving forces.
- The net force F is the vector sum of all the forces acting on an object.

SUMMARY: NEWTON’S LAWS OF MOTION

- Dynamics is the study of forces and the effects the forces have on the velocities of objects.
- The three laws of motion and the SI unit of force are named after Sir Isaac Newton.
- Newton’s first law of motion (also called the law of inertia) states: If the net force acting on an object is zero, the object maintains its state of rest or constant velocity.
- Inertia is the property of matter that tends to keep an object at rest or in motion.
- An object is in equilibrium if the net force acting on it is zero, which means the object is either at rest or is moving at a constant velocity.

SUMMARY: NEWTON'S LAWS OF MOTION

- Newton's second law of motion states: If the external net force on an object is not zero, the object accelerates in the direction of the net force. The acceleration is directly proportional to the net force and inversely proportional to the object's mass. The second law can be written in equation form as $\vec{a} = \frac{\Sigma F}{m}$ (equivalently, $F \ = ma$).
- Both the first and second laws deal with a single object; the third law deals with two objects.
- The SI unit of force is the newton (N): 1 N = 1 kg m/s^2.
- The weight of an object is the force of gravity acting on it in Earth's gravitational field. The magnitude of the gravitational field at Earth's surface is 9.8 N/kg, which is equivalent to 9.8 m/s^2.
- Newton's third law of motion (also called the action-reaction law) states: For every action force, there is a simultaneous force equal in magnitude, but opposite in direction.

PRACTICE

Readings

- Section 2.1, pg 70
- Section 2.2, pg 77

Questions

- pg 76 #1-6
- Pg 87 #1-9